The angular momentum dependence of nuclear optical potentials

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(Dated: February 6, 2013)

Abstract

We discuss the evidence and the arguments for the l-dependence of the nucleon-nucleus and, more generally, the nucleus-nucleus optical model potential. We distinguish general l-dependence, which has not been widely adopted, from parity dependence, a well established property of the interaction between certain light nuclei.

PACS numbers: 25.40.Cm, 24.50.+g, 24.10.Ht, 03.65.Pm

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I. INTRODUCTION

The phenomenological optical model potential is almost always taken to be independent of the partial wave angular momentum, l. However there are both theoretical and phenomenological arguments that some degree of l-dependence is a general property of nuclear optical potentials.

Theory implies that the nucleon-nucleus potential (more generally, the nucleus-nucleus potential) is non-local. Local equivalent potentials that give the same elastic scattering S-matrix, S_l or S_{lj}, (and hence the same elastic scattering observables) can always be found, although the wave functions are different and so are the calculated observables for any reactions in which such potentials play a role. The existence of non-locality is not in doubt, but most phenomenological calculations involving optical model potentials (OMPs), and also the potentials produced by most theoretical calculations, are in terms of such local equivalents. For various reasons, some of which might emerge from the following, l-dependence, and its representation through an l-independent equivalent potential, are by no means as well understood, or even accepted in the way that non-locality is accepted. However, there is no fundamental reason why the interactions between nuclei should not depend upon l, and in the relatively few cases where fully anti-symmetrized calculations have been carried out, there is indeed angular momentum dependence, see Section IV A.

We note a source of occasional terminological confusion: A particular form of l-dependence is *parity dependence* in which the potential takes the form $V_{\mathrm{W}}(r) + (-1)^l V_{\mathrm{M}}(r)$ where the W and M subscripts label the Wigner and Majorana components (which can be complex and contain spin-orbit terms) and l is the partial wave orbital angular momentum. We shall indicate some cases where Majorana terms are rather certain to exist, but in the literature the term 'l-dependent' has sometimes been used for potentials that we always refer to as 'parity-dependent'. In what follows, except when we specifically discuss parity dependence (in Sections II G, IV A), we shall be discussing more general forms of l-dependence.

II. THEORIES OF THE OPTICAL MODEL AND l-DEPENDENCE

Two well-developed theories of the optical model are those due to Feshbach [1] and that, going back to Bell and Squires [2], which is based on the self-energy of a nucleon in nuclear matter. The latter has especially been developed by Mahaux and collaborators [3, 4], see also [5]. As emphasized by Mahaux and Satchler [6] there are quite fundamental differences between these two approaches, not the least being that there is no self-energy theory for composite particles. However, Feshbach's approach has long been a source of insight into the scattering potentials for composite systems. Other theories such as the resonating group model, RGM, have also contributed to our understanding of the interaction between lighter composite nuclei, particularly by exploiting S-matrix-to-potential inversion. RGM and related theories [7] include anti-symmetrization exactly, and reveal information concerning parity dependence as well as more general forms of l-dependence.

A. Feshbach theory

The theory of Feshbach [1] has occasionally been employed in calculations the total contribution to the OMP of all channels coupled to the elastic channel, see e.g. [8–10]. However, this theory more commonly underlies calculations of the contributions of specific selected channels to the OMP, for example in various cases where it is apparent that certain processes are not represented in conventional calculations of the OMP [11–13]. In particular, it can represent processes that vary with nuclear properties in a way that does not enter in the smoothly varying average OMP derived from standard folding models. Such varying contributions are often considered as the 'dynamic polarization potential', DPP, see e.g. [13]. The point we emphasize here is that the formal Feshbach theory leads to explicitly l-dependent and non-local interactions. Nevertheless, local and l-independent representations of the non-local and l-dependent DPP can be found by S-matrix inversion; for a recent discussion see Ref. [14]. In general, local and l-independent potentials representing DPPs exhibit wavy features. Such features can be compared with those found in l-independent potentials that give the same S-matrix S_{lj} as specific known l-dependent potentials. Importantly, local equivalent DPPs are also, invariably, of a form that cannot be represented as a uniform factor multiplying the smooth potentials that arise from local-density folding models. This

fact points to the limited legitimacy of applying uniform normalization factors to folding model potentials.

While it might be plausible that a majority of the very many complex contributions to the full Feshbach OMP somehow average to give an l-independent potential, there remain the contributions to specific strongly coupled channels that vary rapidly from nucleus to nucleus. These typically depend upon the l-transfer appropriate to coupling to the specific states that are strongly coupled to the elastic channel. Arguably, it is the possibility that these couplings do *not* lead to l-dependence that seems implausible. It is unclear to what extent that such l-dependence is of a kind that can be represented by a smoothly varying phenomenological form over a wide range of nuclei and energies.

B. Potentials derived from self-energy

Nucleon-nucleus potentials due to Mahaux and collaborators [3, 4] and their later extensions, Refs. [15, 16], provide a satisfactory, but not precise, fit to nucleon elastic scattering data over a wide range of energies and target nuclei; we refer to these as JLM potentials. The formalism [3, 4] itself presents a local equivalent to the specific non-locality that arises from knock-on exchange, the major source of energy dependence of the JLM potentials. For a given energy, the JLM complex potential $V(r)$ depends on just the proton and neutron densities at radius r: the local density approximation LDA. The original local density model of Ref. [3, 4] was modified (the 'extended local density approximation' of Refs. [15, 16]) in order to correct in a phenomenological way the radial extension of the potential. When applied, this model requires overall normalization factors which vary in a regular way.

Although the local density approximation was 'extended' [3, 4] to correct the radial size of the potential, the model is still a local density model, based only on the proton and neutron densities of the nucleus, and not, for example, the gradient of the density. Specific properties of the nucleus such as its collectivity do not enter nor do any features arising from the interaction of a nucleon with a finite nucleus; there is nothing in the model that corresponds to the orbital angular momentum of the interacting nucleon. The fits to the data do not in general, approach $\chi^2/N = 1$, even with normalization factors. As pointed out above, the effects of channel coupling cannot be represented by a uniform renormalization of the potential. In short, the model leaves room for l-dependent terms.

C. Nuclear Structure Approach

The 'Nuclear Structure Approach' of Vinh Mau [17] and others incorporates some of the physics of the Feshbach approach with the self-energy method. In particular, it includes the effect of coupling to particle-hole states corresponding to giant resonances, and the relationship of this to l-dependence will be mentioned later. The effect of such resonances has been incorporated into optical model studies by Pignanelli *et al* [18] and Delaroche *et al* [19], and see also [20].

D. Possible limits of local density models

Nothing in models based on the local density approximation requires l-dependence in the potential. There is no scope for the angular momentum l of a scattering nucleon to influence a nuclear interaction based on such models in which the finite size of the nucleus and the density gradients in the surface of the nucleus, enter just through the way that the interaction at a particular radius depends on the density at that radius.

Several considerations point to possible weaknesses in the local density model. We know, for example, from the Feshbach model, that collective contributions make a non-local contribution. The excitation of inelastic channels involves the particles in coupled channels propagating in the potential gradients around the nucleus. The coupling leads to non-locality that is distinct from that due to exchange that is represented within the LDA. In Austern's picture [21], flux leaves from the elastic channel at one location and is restored at another location. This second location will, in general have a different local density. In the temporal non-locality discussed by Mahaux and Satchler [6], one can assume that the projectile will return to the elastic channel after it has propagated to a region of different density.

One aspect of the interaction between a nucleon and a finite nucleus that plausibly leads to l-dependence involves the interaction with the density gradient in the nuclear surface. Consider an incident nucleon interacting with a target nucleon in the surface, i.e. in a region of a nuclear density gradient. One can suppose that the reaction on the scattering nucleon will depend upon whether the target nucleon recoils into an increasing or a decreasing nuclear density. This raises the possibility of a term in the potential proportional to $\mathbf{k}\cdot\nabla\rho(\mathbf{r})$

which for a spherical nucleus amounts to an additional potential

$$V_{\mathrm{k}}(r) = \mathbf{k}\cdot\mathbf{r}\frac{\partial\rho(r)}{\partial r}. \tag{1}$$

In these equations $\hbar\mathbf{k}$ is the local nucleon momentum. At high enough energies, where the eikonal approximation is good, $\hbar\mathbf{k}$ may be taken as the incident momentum, in which case a $\mathbf{k}\cdot\nabla\rho(\mathbf{r})$ will make a zero contribution, as can readily be seen by from the eikonal integral for interaction $f(r)\mathbf{k}\cdot\mathbf{r}$:

$$\chi(b) = -\frac{1}{\hbar v}\int_{-\infty}^{\infty} f(r)\mathbf{k}\cdot\mathbf{r}\mathrm{d}z, \tag{2}$$

identifying kb with $l+\frac{1}{2}$ as usual. However, the relationship to l-dependence is immediately apparent with the help of Pythagoras:

$$l^2 = k^2r^2 - (\mathbf{k}\cdot\mathbf{r})^2 \tag{3}$$

which holds when $\hbar k$ is the local momentum. This implies that

$$\mathbf{k}\cdot\mathbf{r} = \pm\sqrt{k^2r^2 - l^2} \tag{4}$$

where the plus and minus signs apply at the outgoing and incident sides of the target nucleus. These effects would not cancel where the projectile is substantially absorbed or where the eikonal approximation fails. If such a term were effective, it would constitute a source of l-dependence. The formulation would be more elaborate if a self-consistently calculated complex local momentum were to be included in a term of the form $\mathbf{k}\cdot\nabla\rho(\mathbf{r})$.

E. Implications of channel coupling

There is now a long history of calculations showing that coupled channels, including reaction channels, make a substantial contribution to elastic scattering and therefore to any local OMP that reproduces the scattering observable. The contribution of low lying vibrations to the proton OMP was studied by Buck [22] and Perey [23] and the contribution of rotational excitations of deformed nuclei was studied in Refs. [11, 12]. For proton scattering, it was found [24, 25] that coupling to deuteron channels by neutron pickup substantially modified the calculated observables, in one case greatly improving the fit for 30.3 MeV protons on ^{40}Ca, a notoriously hard case to fit, see Ref. [26]. In particular a deep minimum in the

angular distribution around 140° was fitted. Later calculations, in which various approximations were lifted, reduced the effect, although the latest study [27] of this case still reveals a substantial DPP arising from the coupling to deuteron channels, although the deep minimum near 140° is no longer fitted. The radial form of this DPP contribution is far from representing a uniform renormalization of the 'bare' (folding model) potential: the real part is repulsive at the nuclear center, with some attraction in the surface, while the imaginary part is absorptive at the nuclear center becoming emissive in the nuclear surface; the real and imaginary spin-orbit terms of the DPP are much wavier than the central terms.

As mentioned in Section II C, Refs [18, 19] studied the effect on proton elastic scattering of coupling to high-lying giant resonances. This coupling led to a good fit to the backward angle minimum for scattering from ^{40}Ca [18] and also from ^{16}O [19]; such coupling should therefore be studied together with pickup coupling. The effect of the giant resonances must be present for all target nuclei, and not just closed shell nuclei ^{16}O and ^{40}Ca in which the deep minima are not filled in by many competing processes. It is therefore desirable to establish the systematic contribution to l-dependence of giant resonance coupling. The giant resonance coupling contribution to the OMP is likely to vary with energy and target nucleus quite differently from the contributions of coupling to low-lying collective states and particle transfer.

F. Identifying coupling effects with l-dependence

In Section III B we shall review the evidence from fitting data for the l-dependence of the nucleon-nucleus interaction. By comparing the effects of neutron pickup coupling [26] upon the elastic scattering S-matrix, i.e. $\arg(S_{lj})$ and $|S_{lj}|$, with the effects on the same S-matrix of the phenomenological l-dependent components, Ref. [28] made the case that l-dependence could be attributed to pickup. It is now straightforward to invert the S-matrix from the l-dependent potential to determine an l-independent representation of the l-dependent term which could be compared with the l-independent representation of the more complete pickup calculations that are now possible.

Delaroche *et al* [19] examined the effect of coupling to giant resonances upon $|S_{lj}|$ but not upon $\arg(S_{lj})$. As shown in [26], it is the argument of the S-matrix which relates most directly to the effect on the real part of the potential. The combination of l-transfer and momentum

transfer involved in exciting giant resonance states is a likely source of l-dependence, and this awaits exploration.

G. The special case of parity-dependence

It has been known for some time that particular exchange processes, particularly in scattering between light nuclei, give rise to parity dependence. This is a consequence of certain exchange terms, including heavy-particle stripping, but not knock-on exchange. These exchange processes are explicitly represented in RGM-GCM calculations. Knock-on exchange is important, leading to non-locality, the local equivalent to which provides most of the energy dependence of the OMP, but does not lead to parity dependence. The influence of other exchange processes was recognized for $n + \alpha$ scattering [29] in calculations that included a Majorana term in the real potential. Subsequently, an imaginary Majorana term was included in an analysis of $p + \alpha$ scattering [30]. These studies involving light target nuclei suggested the application to heavier nuclei, and a real Majorana term was included [31] in an analysis of proton scattering from ^{40}Ca. This work in turn inspired a more extensive exploration [32] of the possible need for Majorana terms in the general nucleon OMP. They found that Majorana terms were important for $p+^{16}$O, less so for a ^{40}Ca target and negligible for scattering from heavier nuclei. These results were supported by the inversion of S_{lj} derived from RGM calculations for proton scattering from nuclei from mass 4 to mass 40 as reviewed in [33], and see below.

Baye [34] has presented theoretical arguments for the variation in strength of parity dependence on the masses of two interacting nuclei. If one is a nucleon, then the Majorana terms are expected to become small as the mass of the target nucleus increases. These predications are borne out by studies of two complementary kinds (see Ref. [33]): i) S-matrix to potential inversion of S-matrices determined by R-matrix and similar fits to scattering data, and (ii) S-matrix to potential inversion of S-matrices arising from RGM calculations. For nucleon scattering from ^{4}He, the same result is found from both approaches: the odd-parity real potential has a volume integral and RMS radius that are substantially greater than those of the even-parity potential. Ref. [33] describes many other cases of nucleus-nucleus scattering, but we simply note here that, as Baye predicted, the strength of the Majorana term for proton scattering does fall off with the mass of the target nucleus, but

is still substantial for nucleon scattering from ^{16}O, as found also in Ref. [32]. However, that work [32] was less decisive being based on imperfect fits to experimental data; these inadequate fits were arguably the result of the omission of other forms of l-dependence.

Parity dependence tends to be associated with the enhancement of the differential cross section at the most backward angles. This can often be identified with the occurrence of heavy particle stripping in the case of nucleon scattering or cluster transfer in the interaction between heavier nuclei. One example of the latter is alpha particle transfer in the case of ^{16}O scattering from ^{20}Ne. For alpha particle scattering from ^{20}Ne, Michel and Reidemeister [35] showed that a small Majorana term markedly improved the fit to elastic scattering angular distributions apparently due to knock-on exchange of the four nucleon cluster. This case exemplifies the problem that occurs in establishing l-dependence: S_l that originates from the parity-dependent potential can always be fitted, by means of $S_l \to V(r)$ inversion [33], with an l-independent potential. In this particular case the l-independent potential was found [36] to be markedly oscillatory and, so, would not have been found by standard angular distribution fitting techniques, in spite of the smallness of the Majorana term. Section IV C presents further discussion of the parity-dependence of interaction potentials between heavier nuclei.

III. IMPLICATIONS OF PHENOMENOLOGY

A. Equivalent representations of l-dependence

Before we discuss the evidence for l-dependence we first acknowledge the particular difficulty in establishing l-dependence in a convincing way. As mentioned above, any S-matrix S_l depending on partial wave angular momentum l, can be subject to $S_l \to V(r)$ inversion [33, 37–39] (or $S_{lj} \to V(r) + \mathbf{l{\cdot}s}\, V_{\mathrm{SO}}(r)$ inversion; from now on, the possibility of spin-orbit inversion is implicit when not stated) leading to an l-independent potential. Thus, even an S-matrix that is generated by an explicitly parity-dependent potential, can be inverted to yield a parity-independent potential. However, the inverted potential in such a case will have oscillatory features ranging from mere waviness to quite marked oscillations. It is important to note that, even if the two potentials (l-dependent and l-independent) yield the same S-matrix, they are certainly not fully equivalent and will, in general, have substantially

different wave functions for $r < R$, where R is the radius outside which the nuclear potential is effectively zero. In many cases it will be apparent that the l-dependent representation is more physical than the oscillatory potential. In principle, any elastic scattering data can be fitted by a local potential, perhaps determined by model-independent fitting (sums of spline functions, gaussian functions, bessel functions etc.) or by fitting the data with an S-matrix followed by inversion of the S-matrix. In that case too, where a potential with wavy features is found, it must be considered likely that an alternative representation, as an l-dependent potential, would be more reasonable. We shall later present examples to illustrate this.

Apart from parity dependence, the great variety of possible forms of l-dependence makes the establishment of l-dependence by fitting experimental data a daunting task. The existence of alternative representations of the potential (wavy or explicitly l-dependent) affords a possible solution to the problem of establishing l-dependence. In principle, model independent fitting should be able to achieve perfect ($\chi^2/\mathrm{DF} \sim 1$) fits, with calculated uncertainties, to observables that have been measured with high precision over a wide angular range. It is known that such precise fits may yield wavy potentials (for deuterons, see Ref.[40], for protons, see Ref.[41]) and 'all' that remains is to establish a correspondence between various forms of waviness and corresponding forms of l-dependence. It is clear that establishing l-dependence, as a signature of the limitations of the local density approach, is of sufficient physical interest that the extraction of the full information content of elastic scattering data is a worthwhile objective. In fact, attempts to extract the full information content of elastic scattering data are now rare, and claims for the 'limitation of the one-channel phenomenological optical model' [42], based on the failure to achieve fits with (visual estimate) $\chi^2/N \sim 20$, are invalid. A failure of Woods-Saxon, WS, potentials to achieve $\chi^2/N \sim 20$, or even $\chi^2/N \sim 1$, is *not* a failure of the phenomenological optical model, but a failure of an unnecessarily restricted form of local potential, which might have been derived from a local-density folding model.

The belief that it is worthwhile to extract the full information content from hard-won, high precision elastic scattering data, appears to be less universal than the belief, commonly expressed in the literature, that a fit with $\chi^2/N \sim 20$ is 'good'. What is unclear, is just how to extract the full information. We have seen that there will always be an l-independent equivalent to any l-dependent potential that gives a precise fit to the data, probably with wavy features, and certainly not of Woods-Saxon form. In fact, there may be many such

potentials, especially when, for proton scattering, the Wolfenstein R-parameter is not fitted, see Ref. [43]. It is also true that we have very little firm knowledge of the topology of the region in parameter space defined by $\chi^2/N = 1$ [44], for data of specific quality. For this reason, such properties of the nucleon-nucleus interaction as its possible l-dependence can not reliably be established from even precise fits for a single nucleus at a single energy: such fits are necessary but not sufficient.

B. Evidence for l-dependence from fits to data

Here we review evidence for l-dependence in nucleon scattering. Sections IV A and IV B discuss two specific forms of l-dependence relevant in particular cases of heavy ion scattering.

In Ref. [45] an l-dependent term was added to an OMP of standard form leading to a substantial improvement to fit to the angular distribution and analyzing power data for 30.3 MeV protons scattering from ^{40}Ca. The data were of unusual precision and of wide angular range and had resisted all attempts to achieve χ^2/N less than about 10. The l-dependent term, which was added to a standard 7-parameter WS plus WS-derivative l-independent central potential, had the following l^2-dependent form:

$$U_l(r) = f(l^2, L^2, \Delta^2)(V_l g_{\mathrm{R}}(r) + \mathrm{i}W_l g_{\mathrm{I}}(r)) \tag{5}$$

where the functions $g_{\mathrm{R}}(r)$ and $g_{\mathrm{I}}(r)$ are standard WS derivative terms (so that the l^2 dependent terms are of surface form), and $f(l^2, L^2, \Delta^2)$ is the standard WS form with L^2 being the 'radius' and Δ^2 the 'diffusivity'. The spin-orbit component had no l-dependent term. The l-dependent potential fitted, in particular, the deep minimum around 140$^{\mathrm{o}}$ in the angular distribution that no WS potential (or folding model potential) has fitted. Ref. [45] compared fits to the data by the l-dependent potentials and the best WS l-independent potential. In Fig 1 we compare the angular distribution and analyzing power as calculated from the l-independent part of the l-dependent potential (dashed line) with the same quantities calculated with the full l-dependent potential (solid line). The substantial change in both quantities due to the l-dependence includes the appearance of a conspicuous minimum near 140$^{\mathrm{o}}$ in the angular distribution, which can be seen to be in agreement with the data; this minimum is not currently understood in terms of interfering amplitudes.

The l-dependent form of Ref. [45] was applied in fits to elastic scattering data for ^{16}O,

FIG. 1: For 30.3 MeV protons on ^{40}Ca, the solid lines are the angular distribution)above) and analyzing power for the l-dependent potential of Ref. [45]. The dashed lines are calculated with the same potentials except that the l-dependent component is omitted; the difference represents the effect of the l-dependent component.

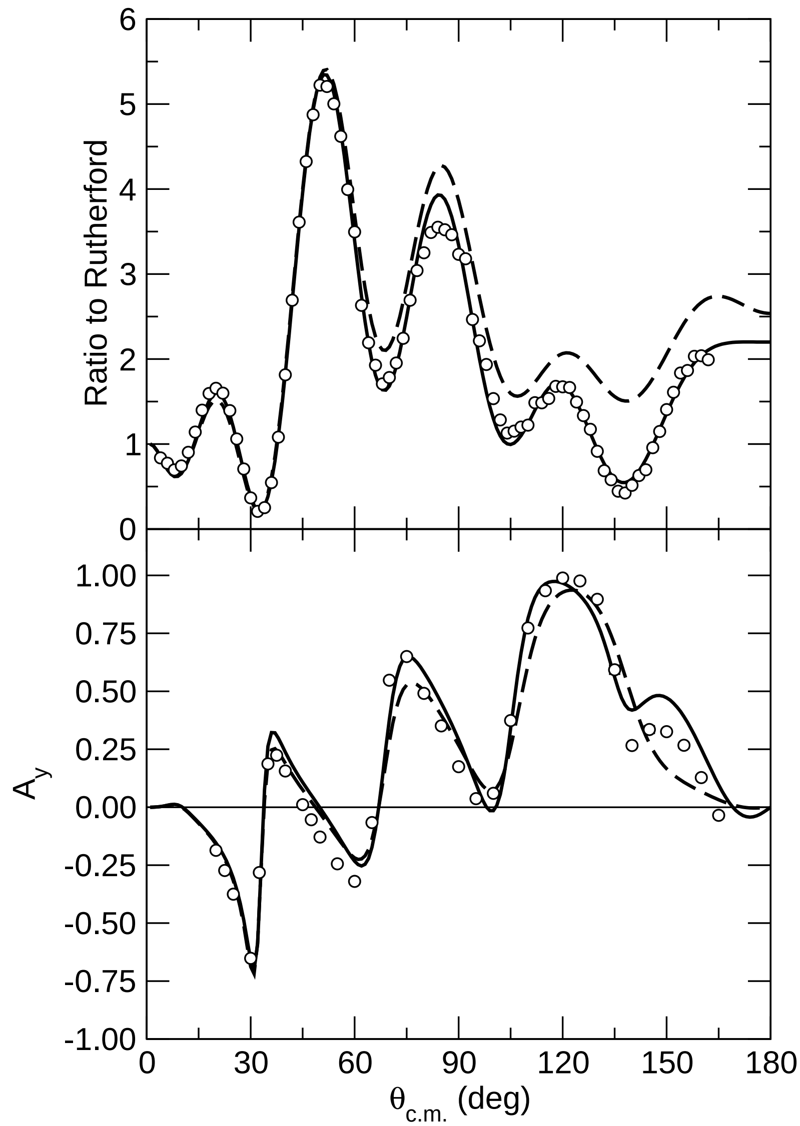
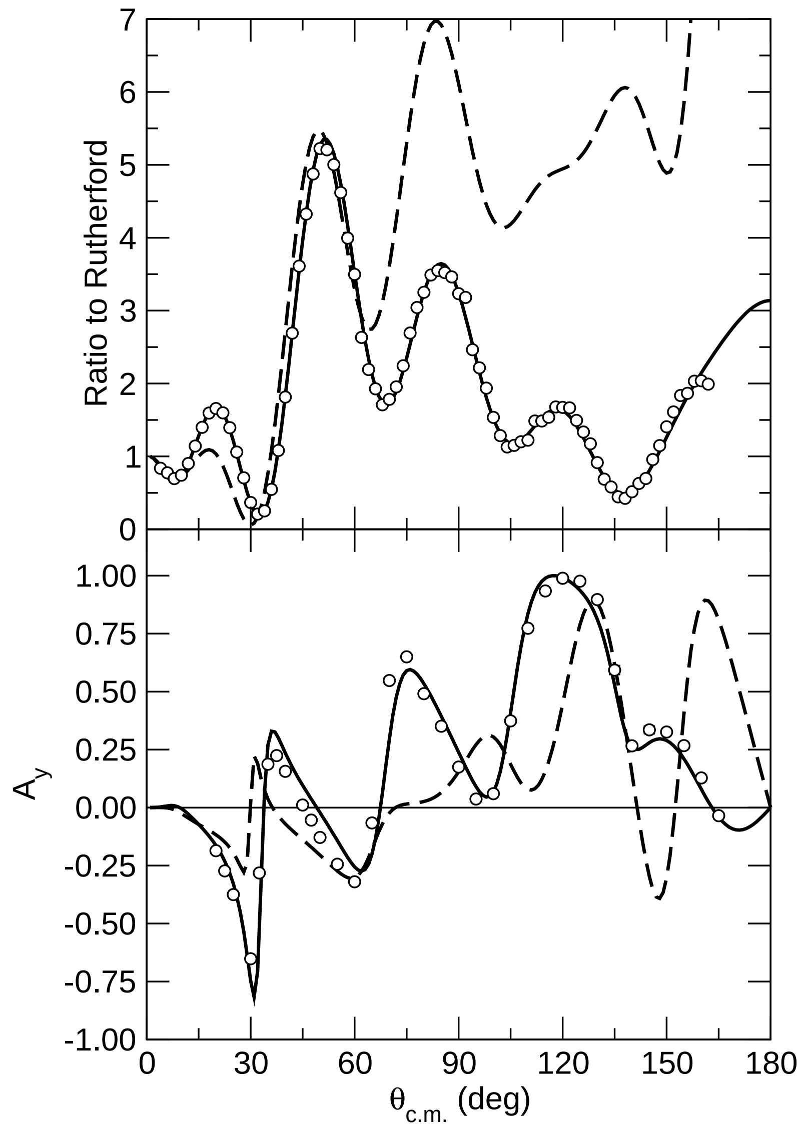


FIG. 2: For 30.3 MeV protons on ^{40}Ca, the solid lines are the angular distribution)above) and analyzing power for the l-dependent potential of Ref. [46]. The dashed lines are calculated with the same potentials except that the l-dependent component is omitted; the difference represents the effect of the l-dependent component.



^{40}Ca, ^{56}Fe and ^{58}Ni and over a wide range of energies in Ref. [46] and applied to further nuclei from ^{15}N to ^{208}Pb in Ref. [47]. In these two papers it was found that good fits over a wide range of energies could be obtained with parameters and with properties (such as volume integrals and rms radii of the l-independent component) that behaved in a more regular fashion than the same properties of best standard l-independent Woods-Saxon fits. There were, in fact, suggestive exceptions in which resonance-like features appeared at certain energies on otherwise smoothly varying quantities. The same quantities for the corresponding best WS fits behaved more irregularly.

The later work of Ref. [46] led to superior fits to both the angular distribution and the analyzing power for 30.3 MeV protons on ^{40}Ca than were found in Ref. [45]. In these later fits, the contribution of the l-dependent terms was very large, as can be seen in Figure 2. Contributions like this were part of a consistent pattern applying for a range of target nuclei and energies.

C. Interpretation of the l-dependence found by fitting data

Two questions arise from the phenomenological l-dependence of the proton OMP:

1. What is the relationship between the l-dependent terms of the fitted l-dependent potentials and the contribution (DPP) generated by the coupling to deuteron channels and evaluated by inverting S_{lj} from CRC calculations?

2. What is the relationship of the fitted l-dependent potentials to the wavy l-independent potentials that are found when model-independent fitting (using splines, gaussian functions, etc) is applied to achieving high quality fits to the same data?

Answers to these questions should illuminate the relationship between the wavy features of the model independent fitted l-independent potentials and the features that emerge from inverting S_{lj} from CRC pickup calculations.

1. Relating l-dependence to the effects of channel coupling

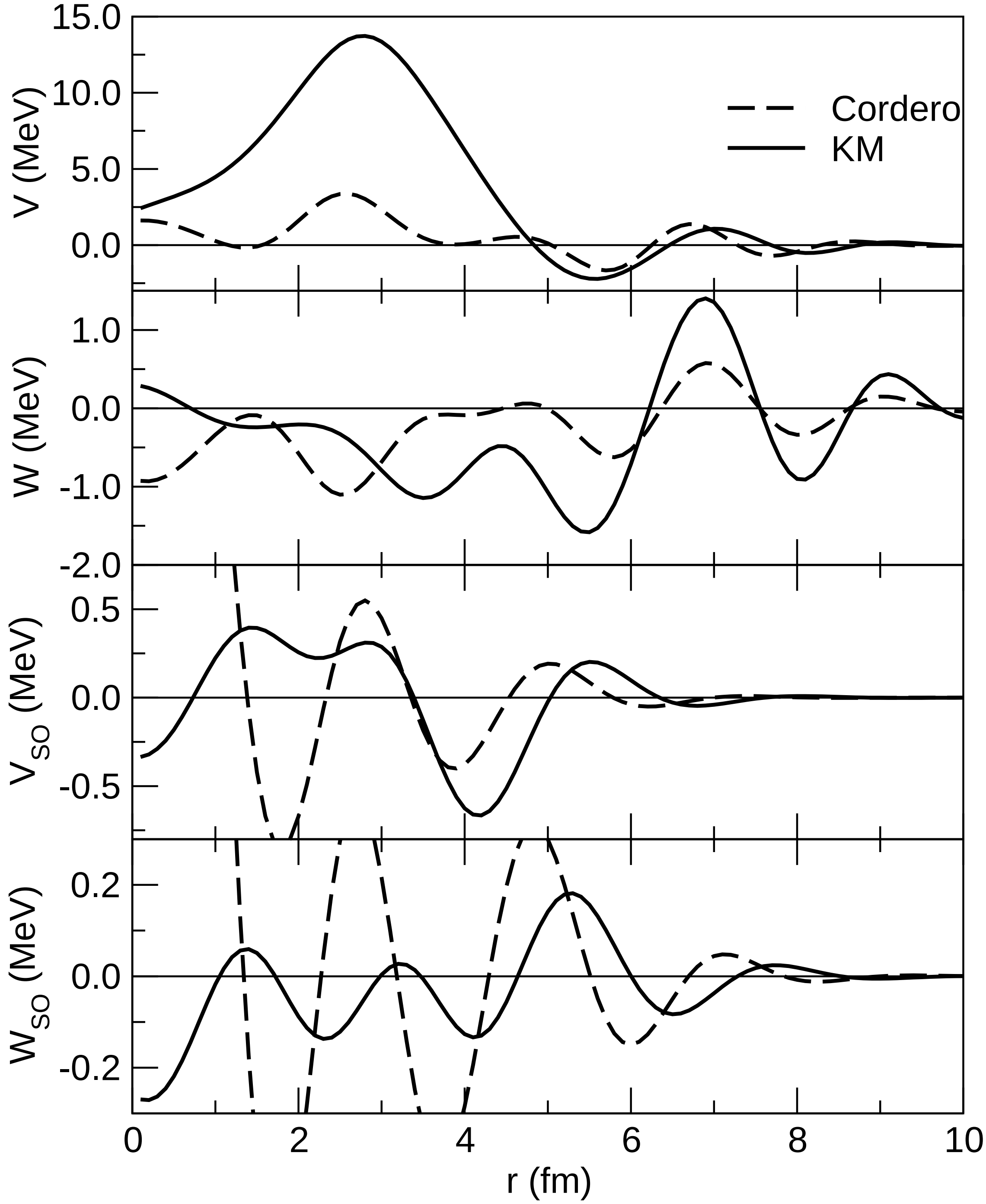
The first question can be approached in two ways: firstly, one can directly compare the changes in $|S_{lj}|$ and $\arg(S_{lj})$ that are due to reaction channel coupling with the contribution

of the l-dependent term to the same quantities. This was done in Ref. [28] where it was shown that there was a qualitative similarity. Secondly, one can invert the S-matrix from the l-dependent potential, and compare the (wavy) features that appear in the inverted l-independent potential, that is found in this way, with the DPPs found by inverting the CRC S-matrix. Having a (somewhat wavy) l-independent equivalent to the l-dependent potential provides a natural means to consider the relationship to potentials found by model-independent fitting of data, and we comment on this below. Figure 3 distills that waviness: the dashed lines were obtained by inverting the S-matrix from the l-dependent potential of Ref. [45] and subtracting the l-independent part of the same potential. In this sense it is a potential representation of the l-dependence of that potential. A conspicuous feature is the appearance of emissive regions in the imaginary central term around 7 fm and 9 fm. These emissive regions persist in the actual complete (unsubtracted) potential. Such local emissivity does not lead to the breaking of the unitarity limit and is a consequence of the fact that local l-independent representations of l-dependent potentials generally have oscillatory features. Apart from the oscillatory features, the overall feature for $r < 5$ fm is a combination of absorption in the imaginary central term and repulsion in the real central term. This combination is a characteristic contribution to the proton-nucleus interaction of the coupling to deuteron channels [27]. The earlier, more approximate CRC calculations [26], did reproduce the minimum near 140$^{\text{o}}$ but the more complete CRC calculations did not [27]. The phenomenological fit suggests that there is some source of l-dependence, and a prime candidate is the coupling to high-lying collective states, see Refs. [18, 19].

The larger effect of the l-dependent term for 30.3 MeV protons on ^{40}Ca that was found by Ref. [46] is reflected in the magnitude of the contribution to the inverted potentials, also presented in Fig. 3 (solid lines). The general features of repulsion and absorption for $r < 5$ fm remain as before, although the repulsion for $r < 5$ fm is markedly increased. The emissive regions in the surface for the potential of Ref. [26] are qualitatively similar to what they were for the earlier fit. In particular, the surface oscillations are in phase and have much the same wavelength. The difference between the dashed and solid lines shows how information can be squeezed from data by pushing for more precise fits to the measured data.

The previously cited work of Refs. [18, 19], demonstrated the large effect of coupling to giant resonances. Such calculations need to be repeated and related to l-dependence in the two ways we have discussed. Firstly, the effect of such coupling can be directly compared

FIG. 3: For 30.3 MeV protons on ^{40}Ca, the four components of the l-independent equivalent of the full l-dependent potential of Ref. [45] (dashed lines) and of Ref. [46] (solid lines) with, in both cases, the l-independent part of that potential subtracted. From the top: the real central, imaginary central, the real spin-orbit and imaginary spin-orbit components.



with the contribution to the S-matrix of the phenomenological l-dependent term; this was done in Ref. [19] for just $|S_{lj}|$ but not $\arg(S_{lj})$, the quantity most directly related to the real part of the potential. Secondly, the S-matrix from the coupled channel calculation should be inverted to determine the DPP due to coupling to giant resonances. This can be compared to phenomenological potentials that fit the data, and also indicate what is missing from folding models (this *not* uniform renormalization).

2. Relating l-dependent and model-independent potentials

Ref. [41] presented l-independent potentials fitted to elastic scattering angular distributions and analyzing powers for protons scattering from ^{16}O and ^{40}Ca for various energies. These model independent fits using spline functions were described as 'theoretically unprejudiced fits' although it is now clear that a prejudice was imposed: the prejudice that the imaginary part of the potential should be absorptive everywhere. It is now understood that this is not a necessary condition for $|S_{lj}| \leq 1.0$ (the unitarity limit) and oscillatory imaginary potentials can have localized emissive regions without breaking the unitarity limit, see below. Moreover, as mentioned, the lack of suitable Wolfenstein (spin rotation) data makes unambiguous theoretically unprejudiced fits formally impossible for proton scattering so that only qualitative comparison with the results of Section III C 1 are meaningful. However, model independent fitting absolutely requires wavy potentials, and the waviness found for the case of ^{40}Ca does share some features with that in Fig. 3, in particular repulsion near 3 fm.

D. The connection between L-dependence and wavy potentials

The DPP arising from specific coupled channels, and determined by inverting the S-matrix from the coupled channel calculation, generally has a somewhat oscillatory character. For many cases of proton scattering, the DPP due to pickup coupling is invariably rather wavy. This waviness can be established to be not an artefact of the inversion procedure and is a general occurrence. For example, following ^{6}Li + ^{12}C CDCC calculations [48], there was a tendency for the local DPP due to breakup of ^{6}Li to be somewhat wavy in the surface for the lowest energy (90 MeV) case. A recent study [49] of deuteron breakup on ^{58}Ni studied the fact that $|S_l|$ often *increases* as a result of processes that might be thought absorptive,

breakup in that case. When that study was extended down to 50 MeV, a quite significantly wavy shape appeared in the surface of the inverted potential. The wiggles seemed to make a nearly zero contribution to the volume integral to the potential.

To get some understanding of these wiggles, we carried out simple calculations for that case, 50 MeV deuterons on ^{58}Ni, posing a quite basic question: what l-dependent modification of an S-matrix derived from an l-independent potential might give rise to such wavy forms? This aspect of potential scattering theory seems to have had little attention. The argument, $\arg S_l = 2\delta_l$, and modulus, $|S_l|$ of the S-matrix $S_l = \exp(\mathrm{i} \arg S_l)|S_l|$ calculated from a standard WS potential were independently modified ($\arg S_l$ and $|S_l|$ relate mostly to the real and imaginary parts of the potential respectively) and the new S-matrix was inverted. We here briefly describe results for modifications such that S_l was unchanged for lowest l and either $|S_l|$ or $\arg S_l$ was modified for high-l, with a smooth transition; a fuller account is available in [50]. In both cases the inverted potential had oscillatory features, these being larger for the real part when $\arg S_l$ was modified and larger for the imaginary part when $|S_l|$ was modified. It is noteworthy that the modification of $\arg S_l$ had a much larger effect on J_{R} than on J_{I} and effectively zero effect on the total cross section although the elastic scattering angular distribution was modified significantly. The modification of $|S_l|$ was such that, $(1 - |S_l|)$ was multiplied by

$$f_{\mathrm{m}}(l) = 1 + z_{\mathrm{m}} \frac{1}{1 + \exp\left((l - l_{\mathrm{m}})/a_{\mathrm{m}}\right)} \tag{6}$$

for $l_{\mathrm{m}} = 14$, $z_{\mathrm{m}} = 0.1$ and $a_{\mathrm{m}} = 2$ with the asymptotic effect:
for $l \ll l_{\mathrm{m}}$, $|S_l| \to |S_l|$,
for $l = l_{\mathrm{m}}$, $1 - |S_l| \to (1 - |S_l|) + \frac{z_{\mathrm{m}}}{2}(1 - |S_l|)$, and
for $l \gg l_{\mathrm{m}}$, we have $1 - |S_l| \to (1 + z_{\mathrm{m}})(1 - |S_l|)$.

The effect of this was to increase J_{I} and the reaction cross section and to induce Fraunhofer-like oscillations on the elastic scattering angular distribution. The effect was linear insofar as, for example, all these effects changed sign for $z_{\mathrm{m}} = -0.1$. The effects most relevant here were found when the modified S-matrix was inverted: (i) very strong oscillations appeared in the imaginary potential, (ii) oscillations also appeared in the real part corresponding to very small changes in the volume integral and rms radius, (iii) the oscillations in the imaginary part in the surface included incursions into emissivity. Of course there was no question that unitarity was broken since the modification of $|S_l|$ specified above

did not allow that.

From the point of view of understanding the equivalence between l-dependence of potentials and the appearance of wavy potentials, point (iii) is particularly significant. It tells us not to exclude, on unitarity grounds, the possibility, when performing model independent data fitting, that the imaginary component has the wrong (emissive) sign in local radial regions. Moreover, we should not expect waviness in just the real or just the imaginary component.

IV. SCATTERING OF HEAVIER NUCLEI

The previous discussion has focussed on nucleon scattering, but new arguments for l-dependence arise for the scattering of composite nuclei. There is a substantial literature relating to the apparently successful application of l-dependence in heavy-ion scattering and there are quite independent theoretical arguments for l-dependence in the real and imaginary components. Results given in sections IV A and IV B suggest that when both the real and imaginary parts of a potential are l-dependent in different ways, the properties of the real and imaginary terms persist in the complete l-independent potential found by inversion.

A. Consequence of antisymmetrization

An example of l-dependence in the real part is provided by the RGM calculations of Wada and Horiuchi [51] for $^{16}O + ^{16}O$. The l-dependence arises from exchange terms going far beyond the 1-particle knock-on exchange, the only exchange terms normally included in folding models. Horiuchi [52] reviews such calculations in the context of a more general discussion of microscopic nucleus-nucleus potentials. There is, of course, no possibility in this case of that other characteristic outcome of including exchange, i.e. Majorana terms, although such terms will arise when the interacting nuclei are not identical bosons. The set of S_l values corresponding to the l-dependent real potentials of Wada and Horiuchi have been inverted [53] to yield an l-independent potential which is significantly different at lower energies from that derived [51] using WKB methods. The difference between the equivalent complete l-independent potential from the l-independent (non-exchange) part of the [51] potential is most marked in the nuclear interior and therefore less significant in the context

of a potential that includes an absorptive term. Nevertheless, this work clearly established that exchange processes lead to an l-dependence of nucleus-nucleus interactions which is in addition both to any parity-dependence and also to contributions arising from knock-on exchange, the only form of exchange ordinarily included in nucleus-nucleus interactions.

The model for ^{16}O + ^{16}O scattering of Kondo *et al* [54], included a phenomenological l-dependent real term inspired by the model of Wada and Horiuchi, together with an l-dependent imaginary term discussed in Section IV B. The S_l for the potential with both terms l-dependent was readily inverted [55] and the resulting real potential had a very similar shape and energy dependence to that found [53] for the Wada-Horiuchi potential.

B. Reduced absorption for high-L

Chatwin *et al* [56] introduced explicit l-dependence into the OMP for heavier ions in which the absorptive term was reduced (smoothly) for the highest partial waves. This was justified, with reference to Feshbach's theory, on the grounds of the reduced number of channels for available for absorption for these partial waves. This approach has had some success and has been applied in various cases, e.g. [57]. It was included in the model of Kondo *et al* [54] mentioned above and it was found, Ref. [55], that except at the highest energy, the l-independent equivalent of the imaginary part had a form radically different from that of any l-independent potential found by fitting data. The imaginary potential of Chatwin *et al* was also included together with a parity-dependent real potential for ^{16}O + ^{20}Ne scattering by Gao and He [58] and the resulting S_l were inverted [59] to produce an l-independent representation. The resulting imaginary potential was qualitatively similar to that produced [55] by the model of Kondo *et al* [54].

Ref. [60] shows how the energy dependence of the l-dependent cutoff of Ref. [56] leads, by way of dispersion relations, to an l-dependence in the real potential for ^{16}O + ^{16}O scattering.

C. Consequences for folding models for heavier ions

It must be supposed that processes that lead to l-dependence for proton scattering should also give rise to l-dependence for heavier nuclear projectiles and this requires phenomenological investigation, particularly in cases where wavy potentials have been found in model

independent fits. This will presumably apply to lighter composite nuclei that are sensitive to more than the nuclear surface.

We note that single folding calculations, based on theoretical nucleon potentials of the kind discussed in Section II B, have been applied with some success [61, 62] to the scattering of lighter composite nuclei. This poses the question as to how an l-dependent nucleon potential should be incorporated in such single folding calculations. We propose that the best procedure, to the extent that l-dependence can be associated with calculable reaction processes, is to include those reaction processes into the scattering calculations for the composite nuclei. For example, if it can be confirmed that the coupling to giant resonance states of the target is a major source of l-dependence for nucleon scattering, then the same processes should be studied in the context of the scattering of composite nuclei. Such processes would become a coupled channel extension of the single folding model calculations.

V. CONCLUSIONS AND DISCUSSION

It is a shame that the full information contained in much high quality elastic scattering data is rarely exploited in a systematic way. There appears to be a prejudice about 'just fitting data'; one may reflect that is was fortunate that Kepler did not feel that way about fitting Tycho Brahe's high quality planet scattering data. We do not yet know how to fully exploit elastic scattering data, and in fact the absence of spin-rotation data is a real problem [43]. Nevertheless, we know that the success of conventional folding models is incomplete as are present attempts [27] to reproduce the data with channel coupling effects, suggesting limits to the local density approximation. At present there are no firm *direct* predictions of l-dependence for proton scattering, as there are for the scattering of heavier nuclei, as we noted in Section IV A, and this is a challenge. It appears that our understanding of nucleon-nucleus scattering is incomplete even at the most phenomenological level.

VI. ACKNOWLEDGMENT

I am grateful to Nicholas Keeley for producing publishable figures.

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[1] H. Feshbach, Ann. Phys. **5**, 357 (1958); Ann. Phys. **19**, 287 (1962).

[2] J.S. Bell and E.J. Squires, Phys. Rev. Lett. **3**, 96 (1959).

[3] J.P. Jeukenne, A, Lejeune, and C. Mahaux, Phys. Rev. C**10**, 1391 (1974); Phys. Rev. C**15**, 10 (1977)

[4] C. Mahaux and R. Sartor, Advances in Nuclear Physics, vol. 20, ed J.W. Negele and E. Vogt (Plenum, New York, 1991), p. 1.

[5] F.A Brieva and J.R. Rook, Nucl. Phys. **A 291**, 299 (1977); Nucl. Phys. **A 291**, 317 (1977); Nucl. Phys. **A 297**, 206 (1978).

[6] C. Mahaux and G.R. Satchler, Nucl. Phys. **A 560**, 5 (1993).

[7] P. Descouvement and M. Dufour, in C. Beck (ed.) *Clusters in Nuclei Vol.2*, Lecture Notes in Physics, vol 848, p. 1 (2012).

[8] C.L. Rao, M. Reeves and G.R. Satchler, Nucl. Phys. **A207**, 182 (1973).

[9] C.A. Coulter and G.R. Satchler, Nucl. Phys. **A 293**, 269 (1977).

[10] G.H. Rawitscher, Nucl. Phys. **A475**, 519 (1987).

[11] N.K. Glendenning, D.L. Hendrie, and O.N. Jarvis, Phys. Lett. **26B**, 131 (1968).

[12] R.S. Mackintosh, Nucl. Phys. **A 164**, 398 (1971).

[13] G.R. Satchler, *Direct Nuclear Reactions* (Clarendon Press, Oxford, 1983).

[14] R.S. Mackintosh and N. Keeley, Phys. Rev. C**81**, 034612 (2010).

[15] E. Bauge, J.P. Delaroche, and M. Girod, Phys. Rev. C**58**, 1118 (1998).

[16] E. Bauge, J.P. Delaroche, and M. Girod, Phys. Rev. C**63**, 024607 (2001).

[17] N. Vinh Mau, *Microscopic Optical Potentials*, Lecture Notes in Physics, ed. H. V. von Geramb (Springer Verlag, New York, 1979) , **89**, p. 104.

[18] M. Pignanelli, H. V. von Geramb, and R. DeLeo, Phys. Rev. C**24**, 369 (1981).

[19] J.P. Delaroche, M.S. Islam, and R.W. Finlay, Phys. Rev. C**33**, 1826 (1986).

[20] G.M. Honoré, W. Tornow, C.R. Howell, R.S. Pedroni, R.C. Byrd, R.L. Walter, and J.P. Delaroche, Phys. Rev. **C 33**, 1129 (1986).

[21] N. Austern, Phys. Rev. B **137**, 752 (1965).

[22] B. Buck, Phys. Rev. **130**, 712 (1963).

[23] F.G. Perey, Phys. Rev. **131**, 745 (1963).

[24] R.S. Mackintosh, Phys. Lett. **B 44**, 437 (1973).

[25] R.S. Mackintosh, Nucl. Phys. **A 230**, 195 (1974).

[26] A.M. Kobos and R.S. Mackintosh, Phys. Lett. **B 62**, 127 (1976).

[27] R.S. Mackintosh and N. Keeley, Phys. Rev. C**85**, 04603 (2012).

[28] R.S. Mackintosh and A.M. Kobos, J. Phys. G: Nucl. Part. Phys. **5**, 359 (1979).

[29] D. R Thompson and Y. C. Tang, Phys. Rev. C**4**, 306 (1971).

[30] D. R Thompson, Y. C. Tang, and R.E. Brown, Phys. Rev. C**5**, 1939 (1972).

[31] G. W. Greenlees, W. Makofske, Y. C. Tang, and D. R. Thompson, Phys. Rev. C**6**, 2057 (1972).

[32] F.K. Vosniakos, N.E. Davison, W.R. Falk, O. Abou-Zeid, and S.P. Kwan, Nucl. Phys. **A 332**, 157 (1979).

[33] R.S. Mackintosh, Scholarpedia 'Inverse scattering: applications in nuclear physics', (2012); arXiv:1205:0468.

[34] D. Baye, Nucl. Phys. **A 460**, 581 (1986).

[35] F. Michel and G. Reidemeister, Z. Phys. A - Atomic Nuclei **333**, 331 (1989).

[36] S.G. Cooper and R.S. Mackintosh, Zeitschrift für Physik A**337**, 357 (1990).

[37] V. I. Kukulin and R. S. Mackintosh, J. Phys. G: Nucl. Part. Phys. **30** R1 (2004).

[38] S.G. Cooper and R.S. Mackintosh, Inverse Problems **5**, 707 (1989).

[39] R.S. Mackintosh, arXiv:1205.0468 (2012).

[40] M. Ermer, H. Clement, P. Grabmayr, G.J. Wagner, L. Friedrich, and E. Huttel, Phys. Lett. B**188**, 17 (1987).

[41] A.M. Kobos and R.S. Mackintosh, Ann. Phys. (NY) **123**, 296 (1979).

[42] R.E. Shamu, J. Barnes, S.M. Ferguson. G. Haouat, and J. Lachkat, J. Phys. G: Nucl. Part. Phys. **17**, 525 (1991).

[43] A.M. Kobos, R.S. Mackintosh, and J.R. Rook, Nucl. Phys. **A 389**, 205 (1982).

[44] R.S. Mackintosh,J. Phys. G: Nucl. Part. Phys. **5**,1587 (1979).

[45] R.S. Mackintosh and L.A. Cordero-L., Phys. Lett **B 68**, 213 (1977).

[46] A.M. Kobos and R.S. Mackintosh, J. Phys. G: Nucl. Part. Phys. **5**, 97 (1979).

[47] A.M. Kobos and R.S. Mackintosh, Acta Physica Polonica, **B** 12, 1029 (1981).

[48] D.Y. Pang and R.S. Mackintosh, Phys. Rev. **C 84**, 064611 (2011).

[49] R.S. Mackintosh and D.Y. Pang, Phys. Rev. **C 86**, 047602 (2012).

[50] R.S. Mackintosh, unpublished manuscript.

[51] T. Wada and H. Horiuchi, Progress of Theoretical Physics, **80**, 488 (1988); **80**, 502 (1988).

[52] H. Horiuchi, *Proc. Int. Conf. on Clustering Aspects of Nuclear Structure and Nuclear Reactions* (Chester, 1984), ed. J.S. Lilley and M.A. Nagarajan (Reidel, Dordrecht), p. 35.

[53] S. Ait-Tahar, R.S. Mackintosh, S.G. Cooper, and T. Wada, Nucl. Phys. **A562**, 101 (1993).

[54] Y. Kondo, B.A. Robson, and R. Smith, Phys. Lett. **B 227**, 310 (1989).

[55] S. Ait-Tahar, S.G. Cooper, R.S. Mackintosh, Nucl. Phys. **A542**, 499 (1992).

[56] R.A. Chatwin, J.S. Eck, D.Robson, and A. Richter, Phys. Rev. C**1**, 795 (1970).

[57] E.L. Reber, K.W. Kemper, P.V. Green, P.L. Kerr, A.J. Mendez, E.G. Myers, and B.G. Schmidt, Phys. Rev. C**49**, R1 (1994).

[58] C. Gao and G. He, Phys. Lett. **B 282**, 16 (1992).

[59] S. Ait-Tahar, R.S. Mackintosh, S.G. Cooper, Nucl. Phys. **A561**, 285 (1993).

[60] S. Ait-Tahar, R.S. Mackintosh, and M.A. Russell, J. Phys. G: Nucl. Part. Phys. **21**, 577 (1995).

[61] D.Y. Pang, P. Roussel-Chomaz, H. Savajols, R.L Varner, and R. Wolski, Phys. Rev. C**79**, 024615 (2009).

[62] D.Y. Pang, Y.L. Ye, and F.R. Xu, Phys. Rev. C**83**, 064619 (2011).